The tempering of glass and the failure of tempered glass plates with pin-loaded joints: Modelling and simulation

Q.D. To a,b,*, Q.-C. He a, M. Cossavella b, K. Morcant b, A. Panait b, J. Yvonnet a

a Université de Marne-la-Vallée, Laboratoire de Mécanique, 5 Boulevard Descartes, F-77454 Marne-la-Vallée Cedex 2, France
b Centre Scientifique et Technique du Bâtiment (CSTB), 84 Avenue Jean Jaurès, 77447 Marne-la-Vallée Cedex 2, France

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Abstract

Tempering is a thermal process to strengthen glass by generating superficial compressive stresses. These residual stresses must be determined and accounted for in the design of tempered glass structures. The present work is concerned with tempered glass plates assembled by pin-loaded joints. In such a structure, a hole in a glass plate is reinforced by a steel ring glued to the glass plate by means of a thin soft resin layer. To determine the residual stresses in a holed tempered glass plate, the tempering process of the latter is first modelled and numerically simulated. In addition, the residual stresses thus determined are then compared with those issued from the photoelasticity measurement. Next, the mechanical behavior of the soft resin is also modelled and experimentally identified. Finally, the failure process of tempered glass plates with pin-loaded joints is analyzed numerically and experimentally, in which unilateral contact, friction, damage and residual stresses are involved. The numerical results obtained by the finite element method turn out to be in good agreement with the experimental ones from real-size tests.

Keywords: Heat treatments; Glass; Mechanical fastening

1. Introduction

Glass is a widely used material owing to its transparency, aestheticism and durability. In terms of its mechanical properties and usage, glass can be classified into two types: float glass and tempered glass. Float glass is easy to produce and widely employed for the manufacture of low load-bearing elements. Because of microdefects, float glass is fragile and vulnerable to stress corrosion attacks [1,2]. The strength of a float glass sheet ranges from 14 to 70 MPa while that of a glass fiber almost free from micron-sized defects is of the order of 3–7 GPa [3]. An effective way to improve the strength of float glass is the thermal tempering which consists in cooling glass suddenly from a liquid state temperature down to the ambient temperature. The tempering process generates residual compressive stresses that strengthen the glass surface whose microdefects are mostly responsible for breakage. Glass that has undergone thermal tempering is called tempered glass. Nowadays, tempered glass is strong and safe enough (see [4–6]) to be used as a structural material to manufacture high load-bearing elements such as beams, columns, floors or facades (see Fig. 1). However, the question remains open on how to determine the strength of tempered glass and to qualify the reliability of structures designed with tempered glass.

From the mechanical point of view, the design of tempered glass structures should be based on their reliability analysis. This analysis is complex for two reasons. First, it necessitates determining the residual stresses of tempered glass due to the thermal tempering which is a quite complicated process. Second, strongly nonlinear phenomena, such as unilateral contact, friction and damage, are involved in
the joint systems used to assemble the elements of tempered glass structures. The objective of the present work is to model and simulate the mechanical behavior of tempered glass structures with reinforced pin-loaded joints. This is a continuation of our previous studies on glass structures with friction-grip joints [7,8] and for pin-loaded joints [9,10] in which no account was taken of residual stress. A typical reinforced pin-loaded joint (see Fig. 2) comprises a fixed part and a moving part. The fixed part is formed of an outer steel ring, a resin layer and a glass plate which are glued together. The moving part is composed of an inner ring and a bolt. In particular, the inner ring helps adjusting the erection axis, the outer ring serves as reinforcement for the glass plate, and the resin acts as a glue material.

The present work makes a complete analysis of tempered glass plates with reinforced pin-loaded joints by performing mechanical modelling, numerical simulation and experimental test. More precisely, to determine the residual stresses in a holed tempered glass plate, the tempering process of the latter is first modelled and numerically simulated. In addition, the residual stresses thus determined are then compared with those issued from the photoelasticity measurement. Next, the mechanical behavior of the soft resin is modelled and experimentally identified. Finally, the failure process of a tempered glass plate with two holes and pin-loaded joints is analyzed numerically and experimentally, in which unilateral contact, friction, damage and residual stresses are involved. The numerical results obtained by the finite element method turn out to be in good agreement with the experimental ones from real-size tests.

2. Glass tempering and residual stress

The tempering of glass is a thermal process consisting in cooling very quickly glass at high temperature (around 600 °C) by air jets. Owing to this process, residual compressive stresses are generated on the surface so as to prevent early failure caused by superficial defects. The tempering modeling and simulations presented in this section aim to obtain the residual stress field of tempered soda-lime-silicate plates with holes.

To model and simulate glass tempering, we need not only thermal tempering parameters data but also the viscous response of glass at different temperatures. The latter can be measured in the laboratory while thermal tempering parameters are much more difficult to obtain directly as they are related to several thermal phenomena like convection, radiation, etc. In the present paper, for simplicity, an effective convection coefficient is proposed and the final results are validated with photoelasticity measurements.

2.1. Thermo-mechanical behavior of glass at high temperature

2.1.1. Viscoelasticity of glass at constant temperature

At normal temperature, glass behaves as an elastic solid. However, at high temperature, for example in a quenching process, glass becomes viscoelastic and its behavior can be described by the following linear viscoelastic relation.

\[
s_{ij}(t) = \int_0^t G_1(t - \tau) \frac{\partial e_{ij}(\tau)}{\partial \tau} d\tau,
\]

\[
\bar{\sigma}(t) = \int_0^t G_2(t - \tau) \frac{\partial \epsilon(\tau)}{\partial \tau} d\tau.
\]

(1)

Fig. 1. A roof system made of tempered glass.

Fig. 2. Composition of a pin-loaded joint.
Above, $s_{ij}$ and $e_{ij}$ are the deviatoric parts of the stress and strain components $\sigma_{ij}$ and $\epsilon_{ij}$, $\bar{\sigma}$ and $\bar{\epsilon}$ are the spherical parts of $\sigma_{ij}$ and $e_{ij}$, which are defined by

$$
\sigma_{ij} = s_{ij} + \bar{\sigma} \delta_{ij}, \quad e_{ij} = e_{ij} + \bar{\epsilon} \delta_{ij}, \quad \bar{\sigma} = \frac{1}{3} \sigma_{ii}, \quad \bar{\epsilon} = \frac{1}{3} e_{ii}.
$$

In Eq. (1), $G_i(t)$ and $G_d(t)$ are two stress relaxation functions. For glass, $G_i(t)$ and $G_d(t)$ can be represented by the Prony series:

$$
G_i(t) = 2G_0 \Psi_1(t), \quad \Psi_1(t) = \sum_{i=1}^{n_1} w_1 e^{-t/\tau_1}, \quad \sum_{i=1}^{n_1} w_1 = 1;
$$

$$
G_d(t) = 3K_\infty + 3(K_0 - K_\infty) \Psi_2(t), \quad \Psi_2(t) = \sum_{i=1}^{n_2} w_2 e^{-t/\tau_2}, \quad \sum_{i=1}^{n_2} w_2 = 1.
$$

The coefficients involved in the Prony series for soda-lime-silicate glass at the reference temperature $T_{\text{ref}} = 864$ K are provided by [5] in Table 1. In our mechanical modelling and numerical simulations, we shall use these numerical values and the following elastic modulus values:

- $E_0 = 70$ GPa, $v_0 = 0.22$,
- $G_0 = E_0/[2(1 + v_0)]$, $K_0 = E_0/[3(1 - 2v_0)]$.

2.1.2. Thermo-rheological simplicity of glass and shift function

To describe the dependence of the viscoelastic behavior on the temperature, the thermo-rheologically simple material supposition principle is used. According to this principle, the forms of the relaxation functions $G_i(t, T = \text{cst})$ at different constant temperatures remain identical when plotted against $\ln t$ (see Fig. 3). More precisely, each function $G_i(t, T = \text{cst})$ can be represented via a function $F_i$ as follows (see, e.g., [11])

$$
G_i(t, T = \text{cst}) = F_i [\ln t - \ln \Phi(T)], \quad i = 1, 2.
$$

Above $\Phi(T)$ is a shift function that verifies $\Phi(T_{\text{ref}}) = 1$ and $G_i(t, T_{\text{ref}}) = F_i [\ln t]$.

There is a rich literature on the study of the shift function $\Phi(T)$, most of which is obtained from the viscosity measurement of materials like glasses, polymers near the transition range (see [12] and the references cited therein). However, the available formulations are all empirical and they are based on the authors’ measured data so that they are different from each other. The most used and widely recognized functions are those of Arrhenius, Vogel–Fulcher–Tamman (VFT) and their varieties like those of Tool–Narayanaswamy–Moynihan (TNM), Adam–Gibbs (AG) [12]. The recent works making use of these models include [5,13,14,16–18]. In the present work, we adopt the Arrhenius equation:

$$
\Phi(T) = \exp \left[ \frac{D}{T - \frac{1}{T_{\text{ref}}}} \right],
$$

According to [5], the value of the material parameter $D$ is equal to 55 000 K for the reference temperature $T_{\text{ref}} = 864$ K.

When the temperature $T$ is not constant but varies, the viscoelastic stress–strain relation becomes [11]

$$
s_{ij}(t) = \int_{0}^{t} G_i(\xi - \xi', T_{\text{ref}}) \frac{d\epsilon_{ij}(\tau)}{d\tau} d\tau,
$$

$$
\bar{\sigma}(t) = \int_{0}^{t} G_d(\xi - \xi', T_{\text{ref}}) \frac{d(\epsilon(\tau) - \epsilon_{th}(\tau))}{d\tau} d\tau.
$$

In these two expressions, $\xi(t)$ and $\xi'(t)$ are the so-called reduced times defined by

$$
\xi(t) = \int_{0}^{t} \frac{d\tau'}{\Phi(T(t'))}, \quad \xi'(t) = \int_{0}^{t} \frac{d\tau'}{\Phi(T(t'))},
$$

and $\epsilon_{th}$ is the thermal strain related to the volume relaxation phenomenon presented below.

2.1.3. Phenomenological volume relaxation model

Consider a glass element which, equilibrated at temperature $T_1$, is cooled suddenly to $T_2$. According to [12], the thermal expansion response comprises two parts (see Fig. 4): an immediate response $\epsilon_{th}(0)$ due to the atomic vibration and a relaxation response $\epsilon_{th}(t)$ where the molecules rearrange towards a new equilibrium state to which the strain $\epsilon_{th}(\infty)$ is associated. This response can be expressed by using the volume relaxation function $M_s(t)$ as follows

$$
\epsilon_{th}(t) - \epsilon_{th}(\infty) = M_s(t),
$$

where $\epsilon_{th}(0) = \epsilon_0(T_2 - T_1)$, $\epsilon_{th}(\infty) = \epsilon_0(T_2 - T_1)$

with $\epsilon_0$ and $\epsilon_0$ being the linear dilatation coefficients in the respective liquid and solid states. In our simulations, we use the numerical values of $\epsilon_0$ and $\epsilon_0$ given by [5,6]: $\epsilon_0 = 25 \times 10^{-6}$ C$^{-1}$ and $\epsilon_0 = 9 \times 10^{-6}$ C$^{-1}$. 

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According to the assumption of Narayanaswamy (see [12,15]), $M_i(t)$ is temperature dependent but verifies the thermo-rheological simplicity condition with the same shift function $\Phi(T)$ as before. The volume relaxation function $M_i(t)$ at the reference temperature $T_{ref}$ is also represented by a Prony series

$$M_i(t, T_{ref}) = \sum_{i=1}^{n_i} w_{3i} e^{-t/\tau_{3i}}, \quad \sum_{i=1}^{n_i} w_{3i} = 1.$$  

(10)

In the present paper, taking $n_3 = 6$, six terms are used and the parameters $w_{3j}$ and $\tau_{3j}$ are specified by Table 2.

For a continuously varying temperature $T(t)$, the formula for the thermal strain is obtained by [12]

$$\epsilon_{th}(t) = \alpha(T(t) - T_0) - (\alpha_0 - \alpha_1) \int_0^t M_i(\zeta - \zeta, T_{ref}) \frac{dT}{dr} \, dr.$$  

(11)

2.1.4. Thermal properties of glass

In the above subsections, the mechanical behavior of glass and its dependence on the temperature are formulated. However, to determine the temperature field in the tempering process, a heat transfer problem must be solved in the first place.

Solving the heat transfer problem requires knowing the material thermal properties and specifying the thermal boundary conditions relative to convection and radiation. The thermal properties of soda-lime-silicate glass are temperature-dependent. Its thermal conductivity $k$ [W/m °C] is modeled by

$$k = 0.975 + 8.58 \times 10^{-4} T$$

(12)

while its specific heat $C_p$ [J/kg °C] is described by

$$C_p = 1.433 + 6.5 \times 10^{-3}(T + 273) \quad \text{if} \quad T > 577 \, ^\circ C,$$

$$C_p = 893 + 0.4(T + 273) - 1.8 \times 10^{-7}(T + 273)^2 $$

(13)

if $T \leq 577 \, ^\circ C$.

The thermal boundary conditions are discussed below.

2.2. Thermal tempering simulation and photoelasticity measurement

2.2.1. Thermal tempering simulation

In this subsection, the thermomechanical behavior of a doubly holed glass plate during the tempering is analyzed. This analysis contains two parts. First, a purely thermal computation is carried out to find the temperature field $T(x, t)$. Then, the temperature $T(x, t)$ is incorporated into the thermo-viscoelastic analysis to calculate the residual stresses. The analysis is realized with the help of MSC MARC [19], a powerful Finite Element Method code.

The temperature field at the beginning of the analysis is taken to be uniform $T(x, 0) = T_0 = 620 \, ^\circ C$ and the temperature of cooling air is $20 \, ^\circ C$. Three equivalent convective coefficients $h_1 = 135 \, W/m^2 \, K$, $h_2 = 115 \, W/m^2 \, K$, $h_3 = 90 \, W/m^2 \, K$ are used respectively for the main, edge and hole surfaces of the plate. The analysis stops when the temperature field over the plate is almost uniform and equal to 20 °C.

As the plate is symmetrical with respect to three orthogonal planes and subjected to symmetrical boundary conditions, the model to be analyzed can be reduced to one-eighth of the plate as in Fig. 5. The mesh of the reduced model is presented in Fig. 6 with the final residual hoop stress field. The plotted coordinate system is a cylindrical one whose origin O is placed at the center of the hole. In particular, the residual hoop stress distributions along the thickness at positions (1a–1b), (2a–2b) and (3a–3b) specified in Fig. 6 are presented in Fig. 7. Positions (1a–1b), (2a–2b) and (3a–3b) correspond to the middle of the plate, the edge and the edge of the hole, respectively.

2.2.2. Photoelasticity measurement

The photoelasticity apparatus used is an epibiascope (see Fig. 8) that allows for measuring the residual stress at any superficial point not too close to the border of the plate. The measurement indicates that the superficial residual stress at point 1a is 122 MPa which is in good agreement with the value of 120 MPa obtained by our numerical simulations presented above.

3. Failure analysis of a tempered glass structure with pin-loaded joints

3.1. Elastic analysis of tempered glass accounting for residual stress

The residual stresses $\sigma_{ij}^{res}$ in tempered glass due to the tempering process have been calculated in the above sec-

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Table 2

<table>
<thead>
<tr>
<th>$i$</th>
<th>$w_{3i}$</th>
<th>$\tau_{3i}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.523 \times 10^{-2}$</td>
<td>$5.965 \times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>$8.205 \times 10^{-2}$</td>
<td>$1.077 \times 10^{-4}$</td>
</tr>
<tr>
<td>3</td>
<td>$1.215 \times 10^{-1}$</td>
<td>$1.362 \times 10^{-3}$</td>
</tr>
<tr>
<td>4</td>
<td>$2.286 \times 10^{-1}$</td>
<td>$1.505$</td>
</tr>
<tr>
<td>5</td>
<td>$2.860 \times 10^{-1}$</td>
<td>$6.747$</td>
</tr>
<tr>
<td>6</td>
<td>$2.265 \times 10^{-1}$</td>
<td>$29.63$</td>
</tr>
</tbody>
</table>
tion. Letting $D_g$ denote the tempered glass domain, the residual stress field $\sigma^r_{ij}$ over $D_g$ is self-equilibrated, so that

$$
\frac{\partial \sigma^r_{ij}}{\partial x_j} = 0 \quad \text{in } D_g, \quad \sigma^r_{ij} n_j = 0 \quad \text{on } \partial D_g,
$$

(14)

where $n_j$ is the outward normal vector on the surface $\partial D_g$ of $D_g$.

Next, when the tempered glass plate is assembled with other structural components (resin layer, steel ring, etc.) and subjected to external loads, the resulting total stress field $\sigma_{ij}$ must satisfy the conditions including the equilibrium equations and boundary conditions

$$
\frac{\partial \sigma_{ij}}{\partial x_j} = f_i \quad \text{in } D_g, \quad \sigma_{ij} n_j = T_i \quad \text{on } \partial D_g
$$

(15)

and the Hooke law

$$
\sigma_{ij} - \sigma^r_{ij} = \lambda \varepsilon^{ext}_{ik} \delta_{ij} + 2\mu \varepsilon^{ext}_{ij} \quad \text{in } D_g
$$

(16)

when the glass plate remains elastic. Above, $f_i$ is the volume force, $T_i$ is the traction vector on the glass plate boundary $\partial D_g$, $\lambda$, and $\mu$ are the Lamé constants for glass, and $\varepsilon^{ext}_{ij}$ is the strain tensor caused by the applied external loads. Because the residual stress field is self-equilibrated as expressed by Eq. (14), we can write...
\[
\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial (\sigma_{ij} - \sigma_{ij}^{\text{res}})}{\partial x_j} \quad \text{in } D_g, \quad (17)
\]
\[
\sigma_{ij}n_j = (\sigma_{ij} - \sigma_{ij}^{\text{res}})n_j \quad \text{on } \partial D_g.
\]

Posing \(\sigma_{ij}^{\text{ext}} = \sigma_{ij} - \sigma_{ij}^{\text{res}}\), it follows from Eqs. (15)–(17) that
\[
\frac{\partial \sigma_{ij}^{\text{ext}}}{\partial x_j} = f_{ij} \quad \text{in } D_g, \quad \sigma_{ij}^{\text{ext}}n_j = T_{ij} \quad \text{on } \partial D_g, \quad (18)
\]
\[
\sigma_{ij}^{\text{ext}} = \lambda \delta_{ij} + 2\mu \epsilon^{\text{ext}}_{ij} \quad \text{in } D_g.
\]

The foregoing analysis shows that, whenever the glass plate is elastic, the total stress field \(\sigma_{ij}\) in it can be calculated in two steps: (i) determine \(\sigma_{ij}^{\text{ext}}\) as the stress field produced only by the applied external loads; (ii) compute \(\sigma_{ij}\) by superposing \(\sigma_{ij}^{\text{ext}}\) to the pre-existing residual stress field \(\sigma_{ij}^{\text{res}}\) obtained in the last section.

### 3.2. Tensile strength of tempered glass

The strength of float glass \(\sigma_{tg}\) depends on the load duration due to the stress corrosion crack phenomena [1,2], the size of the element and the surface finish quality. In the tests performed in [5,6], the average value of \(\sigma_{tg}\) ranges from 31 MPa to 56 MPa.

The failure of tempered glass is governed by the total maximal tensile stress \(\sigma_{\text{max}}\). According to the analysis of the last paragraph, we know that \(\sigma_{\text{max}}\) is the maximal principal stress component associated to the total stress field \(\sigma_{ij}\) obtained by superposing the stress field \(\sigma_{ij}^{\text{ext}}\) due to the external loads to the residual stress field \(\sigma_{ij}^{\text{res}}\) due to the tempering process. Then, the failure criterion of tempered glass is simply given by
\[
\sigma_{\text{max}} \leq \sigma_{tg}. \quad (19)
\]

In the particular case where \(\sigma_{ij}^{\text{ext}}\) and \(\sigma_{ij}^{\text{res}}\) are both uniaxial along the same direction, the criterion (19) amounts to saying that the failure of tempered glass takes place when the maximal tensile stress \(\sigma_{\text{max}}\) due to the external loads reaches the tensile strength \(\sigma_t\) of tempered glass which is equal to the superposition of the residual compressive stresses \(\sigma_{\text{res}}\) to the strength \(\sigma_{tg}\) of float glass. In this case, (19) reduces to
\[
\sigma_{\text{max}} \leq \sigma_t = \sigma_{tg} - \sigma_{\text{res}}. \quad (20)
\]

The tensile hoop strength at the hole edge (3a–3b) is of special interest because there is a stress concentration when loading. In Fig. 7, the residual compressive stress at the point 3b (middle of the edge) is the smallest, so it has the lowest tensile strength. The tensile strength \(\sigma_t\) at the point ranges from \(\sigma_{\text{tmin}} = 31 + 85 = 116 \text{ MPa}\) to \(\sigma_{\text{tmax}} = 56 + 85 = 141 \text{ MPa}\). In the paper, a reference tensile strength 128 MPa for that point is adopted.

### 3.3. Resin and its mechanical behavior

The resin used in bolted connections is a cured mixture of an Epoxy resin and a hardener, which often contains voids. Its mechanical behavior is similar to hardened cement paste: elastic-brittle in traction and elastoplastic in compression because of microcracking.

The uniaxial response of the material is determined by the tests reported in [10], which can be summarized as follows

(i) the resin remains elastic if \(\sigma_t < \sigma < \sigma_{\text{cr}},\)

(ii) when \(\sigma_t\) is reached and \(\epsilon_t > \epsilon > \epsilon_{\text{cr}},\) the resin has a hardening behavior;

(iii) when \(\sigma = \sigma_{\text{cr}}\) or \(\epsilon = \epsilon_{\text{cr}},\) the failure occurs.

Above, \(\sigma_t\) and \(\sigma_{\text{cr}}\) are the initial yielding stress and tensile strength; \(\epsilon_t\) and \(\epsilon_{\text{cr}}\) are the corresponding initial yielding and crushing strains.

A multi-axial model is constructed by combining the Rankine criterion and the Drucker–Prager criterion with a bilinear hardening law (Fig. 9). According to [20], the latter reads
\[
f = a I_1 + \sqrt{J_2} - |\sigma_t|(1 - \sqrt{3} \alpha) \leq 0, \quad \alpha = \frac{\sin \phi}{\sqrt{3(3 + \sin^2 \phi)}}, \quad (21)
\]
where \(I_1\) and \(J_2\) are the first and second principal stress invariants defined by
\[
I_1 = \sigma_1 + \sigma_2 + \sigma_3, \quad J_2 = \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right], \quad (22)
\]
and \(\phi\) is the angle of friction.

According to our experimental results, the material parameter values for the resin are
- elastic properties: \(E = 2500 \text{ MPa}, \nu = 0.22\);
- tensile strength: \(\sigma_t = 11 \text{ MPa}\);
- initial compressive yield stress and strain: \(\sigma_t = -50 \text{ MPa}, \epsilon_t = -0.02\);
- maximum compressive yield stress and corresponding strain: \(\sigma_{\text{max}} = -65 \text{ MPa}, \epsilon_c = -0.03\);
- crushing strain and corresponding stress: \(\epsilon_c = -0.045, \sigma = -58 \text{ MPa}\);
- friction angle and pressure-sensitivity index: \(\phi = 20^\circ, \alpha = 0.11\).

### 3.4. Failure analysis of a tempered glass structure

In the present paper, a structure composed of a 19 mm thick glass plate with two reinforced pin-loaded joints is considered (see Fig. 10). Two opposite increasing displacements are applied on the two bolts until the structure fails. The structure is first studied by two models which are different in the type of interface used. The results obtained with these two models are then compared with the experimental results on the same structure. The two models are implemented by using the finite element code MSC MARC.
All the structural components, i.e. glass plate, bolts, resin, inner and outer rings, are meshed by 3D isoparametric solid elements with 8 nodes. Owing to the fact that the problem is symmetrical with respect to 3 orthogonal planes, each model is further reduced to one-eighth with appropriate boundary conditions (see Fig. 11). The external load is an increasing displacement applied on the head surface of each of the two bolt at a small constant rate.

The mechanical properties of the materials involved in the analysis and other than the resin are provided as follows:

- bolt (stainless steel): $E = 200000$ MPa, $\nu = 0.3$;
- inner ring (copper): $E = 70000$ MPa, $\nu = 0.22$;

Fig. 9. Uniaxial behavior and biaxial tensile failure – compressive yield surface.

Fig. 10. A tempered glass plate with two pin-loaded joints, model and test devices.

Fig. 11. Finite element mesh of the reduced model (view near the joint).
outer ring (stainless steel): $E = 200000$ MPa, $v = 0.3$;
frictional coefficient between stainless steel and polished copper in normal environment: $\mu = 0.2$.

In terms of interface, we considered two models

(i) Model 1: the interface is perfect.
(ii) Model 2: the interface is imperfect. The material near the interface is modeled by a finite thickness layer with a tensile strength lower than the resin. Here its tensile strength is taken to be $\sigma_{cr} = 9$ MPa.

The failure mechanisms of the structure evidenced by the foregoing two models are little different and in good agreement with those from the experimental test. More precisely, it can be described as follows (see Figs. 12 and 13):

- The first crack is initiated in the resin near the interface in the first model and inside the interface layer in the second model at lower load level.
- After the first crack, the cracked zone then develops quickly inside the resin and along the interface, tending to separate the left part from the ring. The right part remains glued to the ring but highly compressed and exhibits irreversible plastic strain.
- The glass becomes heavily stressed and fails when the maximal total stress attains the tensile strength $\sigma_f$ at the middle point 3b of the hole edge. It is noted that the residual stress state and the stress state at point 3b is almost uniaxial and their principal direction almost coincides with the hoop direction so that the criterion (20) can be applied. According to numerical results, given the reference strength $\sigma_f = 128$ MPa, the global failure load is equal to 72 kN in the first model and to 76 kN in the second model, see Fig. 14. These values are quite close to the global failure load 65 kN provided by the experimental test.

![Fig. 12. Resin failure followed by failure in tempered glass.](image1)

![Fig. 13. Cracked zone in the resin and stress concentration in glass.](image2)

![Fig. 14. Numerically determined relation between the applied force and the maximal stress in the glass by the two models.](image3)
3.5. Influence of the frictional coefficient

To study the influence of the friction coefficient between the stainless steel and copper on the failure load, a variation in the coefficient from 0 to 0.4 is considered. Fig. 15 illustrates the inconsiderable dependence of the failure load on the coefficient. This can be explained by the fact that the failure takes place far from the contact zone.

4. Conclusions

In this paper, the failure process of tempered glass structures with pin-loaded joints has been systematically studied by a coupled experimental–numerical method. Firstly, the tempering process is simulated numerically and validated by photoelasticity measures. As a result, the residual stress as well as tensile strength field can be both precisely determined. Secondly, the mechanical characteristics of the resin used in the joint are experimentally identified, and a damage model is proposed for it. Thirdly, a numerical procedure is elaborated to analyze the mechanical behavior of tempered glass structures with pin-loaded joints up to failure. Finally, a real-size structure is experimentally tested and numerically simulated. The experimental and numerical results obtained for this structure are compared. Our present study completes our previous ones [7–10], so that a complete and efficient approach is now available both for the design and optimization of glass structures with friction-grip joints or pin-loaded joints.

References


Fig. 15. Influence of the friction coefficient.